Propagation Characteristics of a Microstrip Line Printed on a General Anisotropic Substrate

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Abstract—An analysis is presented for determining the propagation modes in a microstrip line printed on a substrate having both electric- and magnetic-type general anisotropies. An integral equation is derived for the unknown current distribution on the microstrip line. The kernel of this equation is a complicated 2×2 matrix function of the substrate anisotropy and of the substrate thickness. In order to determine the dispersion relations for the propagating waves, this integral equation is reduced into a finite system of linear equations by employing Galerkin's technique. Numerical results are given for several cases, and the effect of rotating the anisotropy axis in anisotropic substrates is investigated. The proposed method can be employed to compute the propagation characteristics of microstrip lines printed on anisotropic substrates.

I. Introduction

During recent years, there has been a growing interest in using microstrip lines above anisotropic substrates. A practical case is the use of ferrite-loaded microstrip lines to develop nonreciprocal printed-circuit microwave and millimeter-wave devices. Even for the widely used dielectric substrates such as fused silica and alumina, the assumption of isotropy is only an approximation, and substrate anisotropy could have important implications on the operation of microstrip circuits [1]. These effects are expected to be amplified at higher millimeter-wave frequencies.

The behavior of guided modes on the ferrite-filled microstrip line with the magnetization perpendicular to the ground plane has been investigated by Borburgh [2]. In this treatment, the method proposed by Itoh and Mittra [3] for the analysis of microstrip lines on isotropic substrates has been employed. The characteristics of single and coupled microstrips on anisotropic substrates with a diagonal permittivity tensor have been analyzed by Alexopoulos and Krowne [4] by using a quasistatic-mode approach. Several other anisotropic dielectric substrate geometries have been treated in the literature [5], [6]. Hybrid modes have been also analyzed for microstrip lines [7], [8] with anisotropic substrates.

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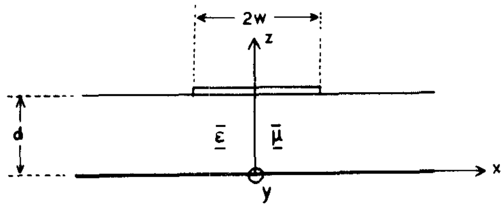

Fig. 1. Anisotropic substrate microstrip geometry.

In this paper, the propagation of guided waves on a microstrip line with the most general—single layer—substrate anisotropy is investigated. The substrate electromagnetic properties are described by the tensors of permittivity $\bar{\epsilon}$ and permeability $\bar{\mu}$. These tensors are defined as 3×3 matrices, and no restrictions are imposed on their elements. Therefore, the presented analysis can be applied easily to any substrate anisotropy of the electric or magnetic type. The microstrip line is assumed uncovered, and therefore the treatment involves a continuous spectrum of eigenwaves. In Section II, an integral equation is developed for the unknown current distribution on the infinitesimally thin microstrip line. In order to determine the characteristics of propagating modes, a moments technique similar to that used in [3] is applied in Section III. Results of the numerical computations are given in Section IV.

In the following analysis, an $\exp(+j\omega t)$ time dependence of the field quantities is assumed and is suppressed throughout.

II. Integral Equation for the Microstrip Line

In Fig. 1, the geometry of a microstrip line printed on an anisotropic substrate is shown. The microstrip width is denoted by $2w$. An anisotropic layer of thickness d is placed on a perfectly conducting plane at $z=0$ (see Fig. 1). The tensors $\bar{\epsilon}$ and $\bar{\mu}$ in a Cartesian coordinate system can be written in the form

$$\bar{\epsilon}=\begin{pmatrix}\epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz}\\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz}\\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz}\end{pmatrix}\quad \bar{\mu}=\begin{pmatrix}\mu_{xx} & \mu_{xy} & \mu_{xz}\\ \mu_{yx} & \mu_{yy} & \mu_{yz}\\ \mu_{zx} & \mu_{zy} & \mu_{zz}\end{pmatrix}. \tag{1}$$

The space above the substrate ($z>d$) is assumed to be occupied by an isotropic and homogeneous medium with

ϵ_0 and μ_0 denoting the permittivity and permeability values, respectively. Then the free-space propagation constant for the $z > d$ half space is defined to be $k_0 = \omega\sqrt{\epsilon_0\mu_0}$.

The solution of the Maxwell equations for a grounded general anisotropic layer has been investigated by Tsalamengas *et al.* [9]. The electromagnetic field $\boldsymbol{E}_a, \boldsymbol{H}_a$ associated with the anisotropic medium can be described in terms of a Fourier integral

$$\begin{matrix}\boldsymbol{E}_a(\boldsymbol{r})\\ \boldsymbol{H}_a(\boldsymbol{r})\end{matrix} = \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y \begin{matrix}\boldsymbol{e}_a(\boldsymbol{k}, z)\\ \boldsymbol{h}_a(\boldsymbol{k}, z)\end{matrix} \exp(-j\boldsymbol{k}\cdot\rho) \tag{2}$$

where $\boldsymbol{k} = k_x\hat{x} + k_y\hat{y}, \rho = x\hat{x} + y\hat{y}$, and the subscript a refers to the anisotropy.

Being interested, in this paper, only in waves guided along the microstrip axis (taken here arbitrarily as the y-axis), it is implied that all the field quantities should have an $\exp(-j\beta y)$ behavior. This means that in (2) the integral over the k_y variable should be reduced to a single term by substituting $k_y = \beta$ or equivalently that the $\boldsymbol{e}_a$ and $\boldsymbol{h}_a$ Fourier transforms have a $\delta(k_y - \beta)$ dependence, with $\delta(\cdot)$ being the delta distribution. Then, following [9, eq. (5)], the electromagnetic field inside the anisotropic substrate can be described in terms of the two-dimensional vectors defined in the Fourier space as

$$\boldsymbol{y}_a(z) = \begin{pmatrix} \boldsymbol{k}\cdot\boldsymbol{e}_a(\boldsymbol{k}, z) \\ \hat{z}\cdot(\boldsymbol{e}_a(\boldsymbol{k}, z)\times\boldsymbol{k}) \end{pmatrix}$$

$$\boldsymbol{x}_a(z) = \begin{pmatrix} \boldsymbol{k}\cdot\boldsymbol{h}_a(\boldsymbol{k}, z) \\ \hat{z}\cdot(\boldsymbol{h}_a(\boldsymbol{k}, z)\times\boldsymbol{k}) \end{pmatrix}. \tag{3}$$

Following a "two-point boundary-value problem" formulation, and by incorporating the boundary conditions on the $z = 0$ perfect conductor surface, the $\boldsymbol{x}_a$ and $\boldsymbol{y}_a$ can be obtained from the relations

$$\boldsymbol{x}_a(z) = \overline{X}_1(z)\cdot\overline{X}_1^{-1}(d)\cdot\boldsymbol{c} \tag{4}$$

$$\boldsymbol{y}_a(z) = \overline{Y}_1(z)\cdot\overline{X}_1^{-1}(d)\cdot\boldsymbol{c} \tag{5}$$

where $\boldsymbol{c}$ is a two-dimensional unknown vector to be eliminated; the 2×2 matrices $\overline{X}_1(z) = [x_{ij}(z)]$ and $\overline{Y}_1(z) = [y_{ij}(z)]$ are given in [9, eqs. (16a)–(17d)].

The electromagnetic field $\boldsymbol{E}_r, \boldsymbol{H}_r$ for the $z > d$ half space can be described in a similar manner as in (2) and (3) by defining new two-dimensional vectors $\boldsymbol{y}_r(z)$ and $\boldsymbol{x}_r(z)$. Then by solving the Maxwell equations for the isotropic region and incorporating the outgoing wave conditions when $z \to \infty$, the following field representation is obtained:

$$\boldsymbol{x}_r(z) = \begin{pmatrix} j\gamma_0 & 0 \\ 0 & \omega\epsilon_0 \end{pmatrix}\begin{pmatrix} F \\ D \end{pmatrix} e^{-\gamma_0(z-d)} \tag{6}$$

$$\boldsymbol{y}_r(z) = \begin{pmatrix} 0 & j\gamma_0 \\ -\omega\mu_0 & 0 \end{pmatrix}\begin{pmatrix} F \\ D \end{pmatrix} e^{-\gamma_0(z-d)} \tag{7}$$

where $\gamma_0 = (k_x^2 + k_y^2 - k_0^2)^{1/2}, \mathrm{Re}(\gamma_0) > 0, \mathrm{Im}(\gamma_0) > 0$, and F, D are unknown expansion coefficients.

Assuming an unknown current distribution $\boldsymbol{J}(\boldsymbol{r}) = e^{-j\beta y}\boldsymbol{J}(x)$ on the microstrip line, the following boundary conditions should be satisfied on the $z = d$ interface plane:

$$\hat{z}x(\boldsymbol{E}_a(\boldsymbol{r}) - \boldsymbol{E}_r(\boldsymbol{r})) = 0 \tag{8}$$

$$\hat{z}x(\boldsymbol{H}_a(\boldsymbol{r}) - \boldsymbol{H}_r(\boldsymbol{r})) = \boldsymbol{J}(\boldsymbol{r}). \tag{9}$$

By transforming these boundary conditions into Fourier space with respect to the x and y variables, the following equations are obtained instead of (8) and (9):

$$\boldsymbol{x}_a(d) - \boldsymbol{x}_r(d) = \boldsymbol{x}_j \tag{10}$$

$$\boldsymbol{y}_a(d) = \boldsymbol{y}_r(d) \tag{11}$$

where

$$\boldsymbol{x}_j = \begin{pmatrix} \hat{z}\cdot(\boldsymbol{k}x\boldsymbol{j}(k_x)) \\ \boldsymbol{k}\cdot\boldsymbol{j}(k_x) \end{pmatrix}\delta(k_y - \beta), \qquad \boldsymbol{k} = k_x\hat{x} + k_y\hat{y} \tag{12}$$

$$\boldsymbol{j}(k_x) = \begin{pmatrix} j_x(k_x) \\ j_y(k_x) \end{pmatrix} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx' e^{jk_x x'}\begin{pmatrix} J_x(x') \\ J_y(x') \end{pmatrix}. \tag{13}$$

Substituting (4)–(7) into (10) and (11) and by eliminating the vector $\boldsymbol{c}$, a solution is obtained for the F, D coefficients in the form

$$\begin{pmatrix} F \\ D \end{pmatrix} = \left[\overline{X}_1(d)\cdot\overline{Y}_1^{-1}(d)\begin{pmatrix} 0 & j\gamma_0 \\ -\omega\mu_0 & 0 \end{pmatrix} - \begin{pmatrix} j\gamma_0 & 0 \\ 0 & \omega\epsilon_0 \end{pmatrix}\right]^{-1}\cdot\boldsymbol{x}_j. \tag{14}$$

Then by substituting (14) into (7), for $z = d$, the electric-field tangential components on the $z = d$ interface plane after a lengthy matrix algebra can be written as follows:

$$\begin{pmatrix} e_{rx}(k_x, d) \\ e_{ry}(k_x, d) \end{pmatrix} = \begin{pmatrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{pmatrix}\begin{pmatrix} j_x(k_x) \\ j_y(k_x) \end{pmatrix}\delta(k_y - \beta) \tag{15}$$

where

$$g_{xx}(k_x, \beta) = \frac{1}{|\boldsymbol{k}|^2}\frac{\det(\bar{x}_1(d))}{\Delta} \times[\omega\mu_0\beta(g_{11}\beta - j\gamma_0 b_1 k_x) + j\gamma_0 k_x(\omega\mu_0 b_4\beta + k_x g_{22})] \tag{16a}$$

$$g_{xy}(k_x, \beta) = \frac{1}{|\boldsymbol{k}|^2}\frac{\det(\bar{x}_1(d))}{\Delta} \times[-(k_x g_{11} + j\gamma_0 b_1\beta)\omega\mu_0\beta + j\gamma_0 k_x(-\omega\mu_0 b_4 k_x + \beta g_{22})] \tag{16b}$$

$$g_{yx}(k_x, \beta) = \frac{-1}{|\boldsymbol{k}|^2}\frac{\det(\bar{x}_1(d))}{\Delta} \times[(\omega\mu_0 k_x(g_{11}\beta - j\gamma_0 b_1 k_x) - j\gamma_0\beta(\omega\mu_0 b_4\beta + k_x g_{22})] \tag{16c}$$

$$g_{yy}(k_x\beta) = \frac{1}{|\boldsymbol{k}|^2}\frac{\det(\bar{x}_1(d))}{\Delta} \times[(g_{11}k_x + j\gamma_0 b_1\beta)\omega\mu_0 k_x + j\gamma_0\beta(-\omega\mu_0 b_4 k_x + \beta g_{22})] \tag{16d}$$

where the terms Δ, $g_{ij}(i=1,2;\ j=1,2)$, and $b_i(i=1,4)$ are defined in the Appendix.

In order to obtain the integral equation which is satisfied by the unknown current distribution on the microstrip line one has to transform (15) into spatial space by using the two-dimensional convolution theorem. Then by imposing the boundary conditions on the perfect conductor microstrip surface, the following integral equation is obtained:

$$\int_{-w}^{w} \bar{G}(x-x')\cdot\begin{pmatrix} J_x(x') \\ J_y(x') \end{pmatrix} dx' = 0 \tag{17}$$

where $-w < x < w$ and

$$\bar{G}(x) = \int_{-\infty}^{+\infty} dk_x \begin{pmatrix} g_{xx}(k_x,\beta) & g_{xy}(k_x,\beta) \\ g_{yx}(k_x,\beta) & g_{yy}(k_x,\beta) \end{pmatrix} \times \exp(-jk_x x).$$

III. Determination of the Propagation Constants β

In order to determine the unknown propagation constants β of the propagating modes on an anisotropic substrate microstrip line, the nontrivial solutions of the homogeneous integral equation (17) should be determined. Following [3], by taking into account the edge conditions at $x=\pm w$, the unknown current distribution on the microstrip line can be described in terms of the series

$$J_x(x) = \sum_{n=0}^{\infty} \alpha_n \sin(n\pi(x+w)/2w) \tag{18}$$

$$J_y(x) = \sum_{n=0}^{\infty} b_n \cos(n\pi(x+w)/2w)\left[1-(x/w)^2\right]^{-1/2} \tag{19}$$

where $|x| < w$.

Applying Galerkin's procedure to (17) with the expansions given in (18) and (19), an infinite system of coupled equations is obtained as follows:

$$\sum_{n=0}^{\infty} \begin{pmatrix} K_{m,n}^{xx}(\beta) & K_{m,n}^{xy}(\beta) \\ K_{m,n}^{yx}(\beta) & K_{m,n}^{yy}(\beta) \end{pmatrix} \begin{pmatrix} \alpha_n \\ b_n \end{pmatrix} = 0, \qquad m = 0,1,2,\cdots \tag{20}$$

where the matrix elements are computed from

$$K_{m,n}^{pq} = \int_{-\infty}^{+\infty} dk_x \Phi_{pm}(-k_x) g_{pq}(k_x,\beta)\Phi_{qn}(k_x) \tag{21}$$

with p and q being equal to x or y and

$$\Phi_{xn}(k_x) = \frac{n\pi}{2w}\,\frac{e^{jk_x w}(-1)^n - e^{-jk_x w}}{k^2 - (n\pi/2w)^2} \tag{22}$$

$$\Phi_{yn}(k_x) = j^n w\left[J_0(k_x w + n\pi/2) + (-1)^n J_0(k_x w - n\pi/2)\right] \tag{23}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function.

The propagation constants β for a given microstrip line can be determined approximately by truncating the infinite system in (20). Then the propagation constants are equal to the roots of the determinant of the truncated finite-order system. However, prior to this, it is necessary to compute numerically the integrals $K_{m,n}^{pq}$ in (21). The integrations are carried out on the real k_x axis. Then it is necessary to take into account the singularities of the integrand functions. It can easily be shown that the singularity points of the integrand functions in (21) are determined from the roots of the equation

$$\frac{\Delta(k_x,\beta)}{\det[\bar{X}_1(d)]} = 0 \tag{24}$$

for a given β value.

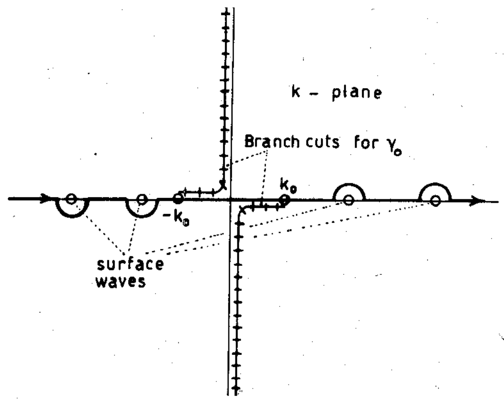

Fig. 2. Complex plane integration contour.

To this end, a numerical root search algorithm has been developed to compute with fine accuracy the location of the roots of (24). It should be noted since $\Delta/\det(\bar{X}_1(d))$ is a complex function, the real and the imaginary parts of this function should vanish for each root. The singularity points correspond to surface waves excited on the grounded anisotropic layer. Assuming the singularities have been determined then in the vicinity of each root, a contour integration is performed by encircling each singularity point by a semi-circle as shown in Fig. 2, where the branch cuts for the γ_0 function are also shown. The contribution from each half circle is equal to the half residue value of the integrand function. A Newton–Cotes with Romberg estimate numerical integration routine is used in the computations. In order to ensure sufficiently high accuracy, lower $(k_x \to -\infty)$ and upper $(k_x \to \infty)$ bounds are taken and a dense subdivision scheme is employed on the real k_x axis.

IV. Numerical Results

In this section, numerical results are given for ferrite and uniaxial sapphire microstrip substrates computed by applying the method described in the previous sections.

In ferrite substrates, the orientation of the biasing magnetostatic field is defined with the unit vector

$$\hat{N} = \cos\vartheta_0\hat{z} + \sin\vartheta_0(\cos\varphi_0\hat{x} + \sin\varphi_0\hat{y}).$$

Then the $\bar{\mu}(\vartheta_0,\varphi_0)$ permeability tensor is computed by applying a unitary transformation to the well-known $\bar{\mu}(\vartheta_0=0,\varphi_0)$ Polder tensor [10]. The notation of [10] is adopted here for the $\bar{\mu}(\vartheta_0=0,\varphi_0)$ tensor elements by

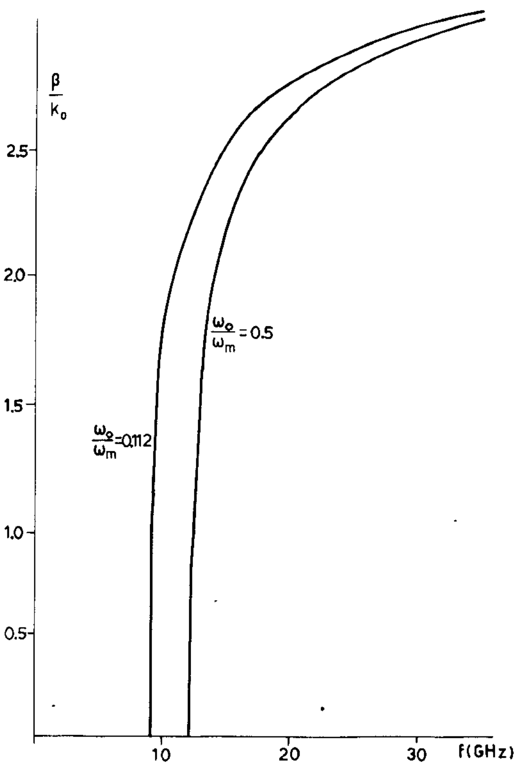

Fig. 3. Normalized propagation constant versus frequency. β/k_0 for $\vartheta_0 = \varphi_0 = 80°$, $d = 0.5$ mm, $w = 1$ mm, and $\epsilon_r = 10$.

TABLE I
COMPUTED β/k_0 VALUES FOR A FERRITE SUBSTRATE AT 20 GHz WITH $\epsilon_r = 10$, $\varphi_0 = 0°$, $d = 1$ mm

W \ ϑ_0	0°	30°	60°	90°
0.50	2.72	2.74	2.79	2.81
1.00	2.80	2.82	2.88	2.91
2.00	2.84	2.88	2.94	2.98
4.00	2.87	2.90	2.97	3.00

TABLE II
PROPAGATION CONSTANTS OF FERRITE-LOADED MICROSTRIP LINES FOR $f = 20$ GHz, $d = 1$ mm, $\vartheta_0 = \varphi_0 = 80°$, AND $\epsilon_r = 10$

W (mm)	β (m^{-1})					
	N=0	N=1	N=2	N=3	N=4	N=5
0.50	1113.43	1115.20	1101.82	1102.09	1101.72	1102.06
1.00	1148.01	1180.57	1138.78	1135.97	1136.05	1136.30

The order of the solution is 0–5 and $w = 0.5$ mm or $w = 1$ mm.

defining the frequencies $\omega_0 = \gamma H_{DC}$, $\omega_m = \gamma M_{DC}$, and ω. The ferrite layer permittivity is taken as a scalar quantity, i.e., $\bar{\epsilon} = \epsilon_0 \epsilon_r \mathbf{1}$. In all of our computations, $\omega_m / 2\pi = 8.408$ GHz (i.e., $M_{DC} = 0.3 \mathrm{Wb/m^2}$).

When we deal with a zero-order solution of the problem, only the terms with $n = 0$ are taken into account for the expansions (18) and (19), and the dispersion equation is simply

$$K_{0,0}^{yy}(\beta) = 0. \tag{25}$$

In an Nth-order solution approach, the summation in (18) and (19) extend over the terms $n = 1, \cdots, N$ and $n = 0, 1, \cdots, N-1$, respectively, leading to a dispersion equation of the form

$$\det \left[\begin{array}{ccc|ccc} K_{1,1}^{xx} & \cdots & K_{1,N}^{xx} & K_{1,1}^{xy} & \cdots & K_{1,N-1}^{xy} \\ \vdots & & \vdots & \vdots & & \vdots \\ K_{N,1}^{xx} & \cdots & K_{N,N}^{xx} & K_{N,0}^{xy} & \cdots & K_{N,N-1}^{xy} \\ \hline K_{0,1}^{yx} & \cdots & K_{0,N}^{yx} & K_{0,0}^{yy} & \cdots & K_{0,N-1}^{yy} \\ \vdots & & \vdots & \vdots & & \vdots \\ K_{N-1,1}^{yx} & \cdots & K_{N-1,N}^{yx} & K_{N-1,0}^{yy} & \cdots & K_{N-1,N-1}^{yy} \end{array} \right] = 0. \tag{26}$$

In order to determine the roots of (26), the determinant is computed for many β values. In a first step, the roots are located roughly and then a Regula–Falsie procedure is applied to determine accurately these roots.

In Fig. 3, the variation of the β/k_0 normalized propagation constant with the operation frequency is given for $\varphi_0 = \vartheta_0 = 80°$ inclined biasing field for two different ω_0/ω_m ratios. Since at higher frequencies the ferrite anisotropy diminishes as ω^{-2}, the difference between the two dispersion curves is very small at 35 GHz. In Table I, results are given for β/k_0 values at 20 GHz for $w = 0.50$–4.00 mm, $\omega_0/\omega_m = 0.1124$, $\epsilon_r = 10$, $d = 1$ mm, $\varphi_0 = 0°$, and several ϑ_0 angles. The variation of the propagation constants with the ϑ_0 angle is larger for wider strips. Usually, one or two surface waves are encountered in the computation of the $K_{m,n}^{pq}$ integrals.

The convergence of the procedure outlined above was found to be fast. Table II includes the values of β corresponding to solutions of order 0–5 for $w = 0.5$ mm and $w = 1.00$ mm.

Furthermore, in order to have an independent check with previously published results, propagation constants in uniaxially anisotropic sapphire substrates have been computed for several frequencies and directions of the anisotropy axis. In this case, the tensors $\bar{\epsilon}$ and $\bar{\mu}$ have the forms

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_1 \cos^2 \vartheta_0 + \epsilon_2 \sin^2 \vartheta_0 & 0 & (\epsilon_1 - \epsilon_2) \sin \vartheta_0 \cos \vartheta_0 \\ 0 & \epsilon_1 & 0 \\ (\epsilon_1 - \epsilon_2) \sin \vartheta_0 \cos \vartheta_0 & 0 & \epsilon_1 \sin^2 \vartheta_0 + \epsilon_2 \cos^2 \vartheta_0 \end{bmatrix} \tag{27}$$

and

$$\bar{\mu} = \mu_0 \bar{\mathbf{1}}_3$$

respectively. The anisotropy axis is defined by the ϑ_0 angle. The parameters ϵ_1 and ϵ_2 are given the values $\epsilon_1 = 9.4$ and $\epsilon_2 = 11.6$, taken from [8]. The geometrical parameters of the structure are taken to be $d = 0.5$ mm and $w = 0.5$ mm. Fig. 3 of [8] illustrates the variation of $\epsilon_{\mathrm{eff}} = (\beta/k_0)^2$ versus $f = \omega/2\pi$ for the structure under consider-

TABLE III
β/k_0 versus F (GHz) AND ϑ_0 FOR A UNIAXIALLY ANISOTROPIC SUBSTRATE ($d = w = 0.5$ mm, $\epsilon_1 = 9.4$, $\epsilon_2 = 11.6$)

ϑ_0 \ f	5	10	20	30
0°	2.91	2.93	3.03	3.09
30°	2.86	2.88	2.96	3.03
60°	2.75	2.77	2.84	2.90
90°	2.69	2.71	2.77	2.83

ation in the special case $\vartheta_0 = 0$. The curve, corresponding to the value $w/d = 2$ of this figure, is closely related to the first row of our Table III (note that the width of the strip in [8] is denoted by w, whereas in our treatment it is noted by $2w$). Comparison between these two reveals an excellent agreement.

V. Conclusions

In this paper, a semi-analytical technique is presented for the analysis of propagating waves on microstrip lines printed on the most general-type anisotropic substrates. Numerical results are given for magnetically or electrically anisotropic microstrip lines.

Appendix

Definitions of the b_i, g_{ij}, and Δ terms appearing in (16a)–(16d) are

$$b_1 = x_{11}(d)y_{22}(d) - y_{21}(d)x_{12}(d)$$
$$b_4 = y_{11}(d)x_{22}(d) - y_{12}(d)x_{21}(d)$$
$$g_{11} = \omega\epsilon_0 \det(\overline{Y}_1(d)) + j\gamma_0 b_2$$
$$g_{22} = -\omega\mu_0 b_3 + j\gamma_0 \det(\overline{Y}_1(d))$$
$$b_2 = y_{21}(d)x_{22}(d) - y_{22}(d)x_{21}(d)$$
$$b_3 = y_{12}(d)x_{11}(d) - y_{11}(d)x_{12}(d)$$

where the elements of the following matrices

$$\overline{X}_1(z) = \begin{pmatrix} x_{11}(z) & x_{12}(z) \\ x_{21}(z) & x_{22}(z) \end{pmatrix}$$

$$\overline{Y}_1(z) = \begin{pmatrix} y_{11}(z) & y_{12}(z) \\ y_{21}(z) & y_{22}(z) \end{pmatrix}$$

are determined by [9, eqs. (16.a)–(17.d)]

$$\Delta = j\gamma_0\omega\epsilon_0 b_1 b_4 - (j\gamma_0 b_2 + \omega\mu_0 \det(\overline{X}_1(d)))(\omega\epsilon_0 b_3 - j\gamma_0 \det(\overline{X}_1(d)))$$

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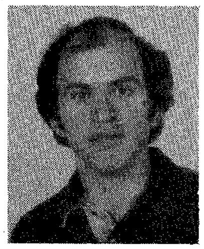

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